

nag_arcsinh (s11abc)

1. Purpose

nag_arcsinh (s11abc) returns the value of the inverse hyperbolic sine, $\text{arcsinh } x$.

2. Specification

```
#include <nag.h>
#include <nags.h>

double nag_arcsinh(double x)
```

3. Description

The function calculates an approximate value for the inverse hyperbolic sine of its argument, $\text{arcsinh } x$.

For $|x| \leq 1$ the function is based on a Chebyshev expansion.

For $|x| > 1$

$$\text{arcsinh } x = \text{sign } x \times \ln \left(|x| + \sqrt{x^2 + 1} \right).$$

This form is used directly for $1 < |x| < 10^k$, where $k = n/2 + 1$, and the machine uses approximately n decimal place arithmetic.

For $|x| \geq 10^k$, $\sqrt{x^2 + 1}$ is equal to $|x|$ to within the accuracy of the machine and hence we can guard against premature overflow and, without loss of accuracy, calculate

$$\text{arcsinh } x = \text{sign } x \times (\ln 2 + \ln |x|)$$

4. Parameters

x

Input: the argument x of the function.

5. Error Indications and Warnings

None.

6. Further Comments

6.1. Accuracy

If δ and ϵ are the relative errors in the argument and the result, respectively, then in principle

$$|\epsilon| \simeq \left| \frac{x}{\sqrt{1+x^2} \text{arcsinh } x} \delta \right|.$$

That is, the relative error in the argument, x , is amplified by a factor at least

$$\frac{x}{\sqrt{1+x^2} \text{arcsinh } x}$$

in the result.

The equality should hold if δ is greater than the **machine precision** (δ due to data errors etc.), but if δ is simply due to round-off in the machine representation, it is possible that an extra figure may be lost in internal calculation round-off.

It should be noted that this factor is always less than or equal to one. For large x we have the absolute error in the result, E , in principle, given by

$$E \sim \delta.$$

This means that eventually accuracy is limited by **machine precision**.

6.2. References

Abramowitz M and Stegun I A (1968) *Handbook of Mathematical Functions* Dover Publications, New York ch 4.6 p 86.

7. See Also

None.

8. Example

The following program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

8.1. Program Text

```
/* nag_arcsinh(s11abc) Example Program
 *
 * Copyright 1989 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */
#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

main()
{
    double x, y;

    Vprintf("s11abc Example Program Results\n");
    Vscanf("%*[^\n]s"); /* skip the first input line */
    Vprintf("      x          y\n");
    while (scanf("%lf", &x) != EOF)
    {
        y = s11abc(x);
        Vprintf("%12.3e%12.3e\n", x, y);
    }
    exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
s11abc Example Program Data
-2.0
-0.5
1.0
6.0
```

8.3. Program Results

```
s11abc Example Program Results
      x          y
-2.000e+00 -1.444e+00
-5.000e-01 -4.812e-01
 1.000e+00  8.814e-01
 6.000e+00  2.492e+00
```
